Third-Order Passive Load Identification Under Non-Sinusoidal Conditions

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Abstract

This paper presents an extension of the well known load identification method, valid under non-sinusoidal conditions, which makes use of a 2nd-order passive circuit and an auxiliary voltage or current generator. The proposed solution is similar, but uses a 3rd-order passive circuit. This allows to identify the passive circuit components with positive or negative parameters, according to the aim of the identification (load modeling or compensation). Moreover, the proposed approach keeps the orthogonality between current/voltage components and removes the indetermination which occurs, with the 2nd-order approach, in the case of sinusoidal operation. As an application example, the proposed approach is applied to the design of a hybrid compensation system including active and passive filtering.

1 Introduction

Load identification and compensation techniques have always been challenging issues, widely discussed in several studies, starting from Shepherd and Zakikhani [1], Klinger [2], Kusters and Moore [3], Page [4], and more recently Czarnecki [5–9], Willems [10] and others. Several studies have been devoted to the identification and design of simple [4–5] or complex [9] passive compensators, which increase the load power factor by compensating – either partially or totally – the load non-active currents, as defined by the Fryze decomposition [11]. Guidelines for the design of hybrid compensators have also been derived, which minimize the requirements of the active part of the filter.

This paper reconsiders the proposed theories, showing that some of the existing approaches can be easily reformulated by making use of suitable power-like quantities, which are conserved in any networks. Using the proposed framework, the load is firstly represented by a parallel connection of a resistance, two reactive elements (an inductance and a capacitance) and a residual current generator and the parameters of the passive elements are identified so as to minimize the rms value of the residual current generator, as reported by Page [4]. This identification, which we refer to as 2nd-order identification due to the presence of two reactive elements, is then extended to the dual configuration given by the series connection of a resistance, an inductance, a capacitance and a residual voltage generator.

The paper then proposes how to overcome the two relevant limitations affecting the 2nd-order identification, i.e. the indetermination of the solution under sinusoidal conditions and the uncontrolled sign of the resulting parameters. These results are achieved by making use of a resistor, three reactive elements and a residual generator, which we refer 3rd-order identification. The identification of the reactive bipoles are obtained by reducing to zero the proposed power-like quantities using only passive elements, thus minimizing the rms value of the residual current or voltage generator within the proposed passive structures. Besides the control of the sign of the reactive parameters, this approach works properly under sinusoidal conditions, bringing to the usual active/reactive decomposition, and conserves the property of orthogonal decomposition of voltage and currents under non-sinusoidal conditions.

2 Basic definitions

Given any two periodic functions \( x(t) \) and \( y(t) \) of period \( T \), we define their “average internal product” and the norm of \( x(t) \) as:

\[
\langle x, y \rangle = \frac{1}{T} \int_{0}^{T} x(t) y(t) dt \quad \| x \| = \sqrt{\langle x, x \rangle}
\]  

(1)

Given a periodic function \( x(t) \) of period \( T \) and an angular frequency \( \omega = 2\pi/T \) we define the derivative operator “tilde” (\( \tilde{\quad} \)) and the integral operator “hat” (\( \hat{\quad} \)) as:

\[
\tilde{x} = \frac{1}{\omega} \frac{dx}{dt} \quad \hat{x} = \omega \left( \int_{0}^{T} x dt - \int_{0}^{T} x dt \right)
\]  

(2)

Note that \( x, \tilde{x}, \hat{x} \) are homogeneous quantities. Moreover:

\[
\tilde{x} = \tilde{\tilde{x}} = x \quad \langle x, \tilde{x} \rangle = \langle x, \hat{x} \rangle = 0 \quad \langle \tilde{x}, \hat{x} \rangle = -\|x\|^2
\]  

(3)

From eq. (3) we can also derive:

\[
\langle \tilde{x}, x \rangle = -\|x\|^2 \quad \langle \tilde{x}, \tilde{x} \rangle = -\|x\|^2
\]  

\[
\langle x, \tilde{x} \rangle = -\langle x, y \rangle
\]  

(4)
Finally, it is worth noting that:
- the definition of derivative term $\dot{x}$ requires that variable $x$ is a continuous function of time $t$
- $X \leq x \leq \bar{X}$ where $\bar{X} = \|X\|$, $X = \|x\|$, and $\bar{X} = \|\dot{X}\|
- under sinusoidal conditions:
  \[x + \dot{x} = 0, \quad X = \bar{X} = \bar{X}\]

3 Power terms

3.1 Application of Tellegen’s theorem

For a given network $\Pi_L$ including $L$ branches, let $u_i$ be the vector of branch voltages $u_i$ and $i$ the vector of branch currents $i_i$. Since voltages $u_i$ and currents $i_i$ are consistent with the network, i.e. they comply with the corresponding Kirchhoff laws, the Tellegen’s theorem ensures that:

\[u \cdot i = \sum_{l=1}^{L} i_l = \sum_{l=1}^{L} p_l = 0\]  \hspace{1cm} (5)

$p_l$ being the instantaneous power absorbed by the $l$-th branch.

Observe now that vector $\dot{u}$ of the derivative voltage terms and vector $\ddot{u}$ of the integral voltage terms are also consistent with network $\Pi_L$ like current vectors $\dot{i}$ and $\ddot{i}$ are. Thus, the Tellegen’s theorem ensures that all following equations are valid for any network $\Pi_L$:

\[u \cdot i = u \cdot \dot{i} = u \cdot \ddot{i} = 0\]  \hspace{1cm} (6)

\[\ddot{u} \cdot i = \ddot{u} \cdot \dot{i} = \ddot{u} \cdot \dddot{i} = 0\]  \hspace{1cm} (7)

\[\dddot{u} \cdot i = \dddot{u} \cdot \dot{i} = \dddot{u} \cdot \dddot{i} = 0\]  \hspace{1cm} (8)

3.2 Average Power Terms

Eqs. (6)–(8) show nine different instantaneous power-like terms (in the sense that they are measured in VA) which comply with the conservation law in any electrical networks. In our approach we make reference to average power terms only. In this instance, only five out of the above nine power terms – are independent, and only three of them are relevant for our approach. Given periodic voltage $u$ and current $i$ at a pair of terminals of a given network, we consider the following average power terms:

\[P = \langle u, i \rangle = -\langle \dddot{u}, \dot{i} \rangle = -\langle \dddot{u}, i \rangle\] \hspace{1cm} (9)

\[\dot{P} = \langle u, \dot{i} \rangle = -\langle \dddot{u}, \dot{i} \rangle\] \hspace{1cm} (10)

\[\ddot{P} = \langle \ddot{u}, i \rangle = -\langle \dddot{u}, \dot{i} \rangle\] \hspace{1cm} (11)

The first quantity is the active power, while terms $\dot{P}$ and $\ddot{P}$ are power-like quantities. Both $\dot{P}$ and $\ddot{P}$ coincide with reactive power $Q$ under sinusoidal operation.

Note that, according to the Tellegen’s theorem, quantities $P, \dot{P}, \ddot{P}$ are conserved in any network $\Pi_L$, i.e.:

\[\sum_{l=1}^{L} p_l = \sum_{l=1}^{L} \dot{p}_l = \sum_{l=1}^{L} \ddot{p}_l = 0\] \hspace{1cm} (12)

where $p_l, \dot{p}_l, \ddot{p}_l$ are the power terms associated to the generic $l$-th branch of the network.

3.3 Power terms of elementary bipoles

Power terms $P, \dot{P}$ and $\ddot{P}$ are computed hereafter for purely resistive, inductive and capacitive bipoles.

Given a resistor of resistance $R$ and conductance $G = 1/R$, under periodic operation we have:

\[u = R \dot{i} \leftrightarrow \dot{i} = G u \Rightarrow U = R I \leftrightarrow I = G U\] \hspace{1cm} (13)

\[P = \langle u, i \rangle = R \langle \dot{i}, i \rangle = R I^2 = G \langle u, u \rangle = G U^2\] \hspace{1cm} (14)

\[\dot{P} = \langle u, \dot{i} \rangle = R \langle \dot{i}, \dot{i} \rangle = 0\] \hspace{1cm} (15)

\[\ddot{P} = \langle \ddot{u}, i \rangle = G \langle \ddot{u}, u \rangle = 0\] \hspace{1cm} (16)

Given an inductor of inductance $L$ and invariance $L = 1/L$, under periodic operation we have:

\[u = \omega \dot{L} \leftrightarrow \dot{L} = \omega U \Rightarrow L \dot{i} \leftrightarrow i = \omega U\] \hspace{1cm} (17)

\[\dot{P} = \langle u, \dot{i} \rangle = \omega L \langle \dot{i}, \dot{i} \rangle = \omega L I^2\]
\[= -\langle \dddot{u}, \dot{i} \rangle = -\frac{L}{\omega} \langle \dddot{u}, \dddot{u} \rangle = \frac{L}{\omega} U^2\] \hspace{1cm} (18)

\[\ddot{P} = \langle \dddot{u}, i \rangle = \omega L \langle \dddot{u}, i \rangle = \omega L I^2 = \langle u, \dot{i} \rangle\]
\[= -\omega L \langle \dddot{i}, \dot{i} \rangle = \omega L I^2\] \hspace{1cm} (19)

Note that, in this case, $\dot{P}$ and $\ddot{P}$ are positive defined quantities, while $P$ is zero, of course. Under sinusoidal operation $P$ and $\dot{P}$ are equal and coincide with the inductor reactive power.

Given a capacitor of capacitance $C$ and elastance $\Gamma = 1/C$, under periodic operation we can write:

\[u = \frac{\Gamma}{\omega} \cdot \dot{i} \leftrightarrow \dot{i} = \omega C \dot{u} \Rightarrow U = \frac{\Gamma}{\omega} \dot{I} \leftrightarrow I = \omega C U\] \hspace{1cm} (20)

\[\dot{P} = -\langle \dddot{u}, \dot{i} \rangle = -\omega C \langle \dddot{u}, \dddot{u} \rangle = -\omega C U^2 = \langle u, \dot{i} \rangle\]
\[= \frac{\Gamma}{\omega} \langle \dddot{i}, \dot{i} \rangle = -\frac{\Gamma}{\omega} I^2\] \hspace{1cm} (21)

\[\ddot{P} = -\langle \dddot{u}, i \rangle = -\frac{\Gamma}{\omega} \langle \dddot{u}, \dot{i} \rangle = -\frac{\Gamma}{\omega} I^2\]
\[= \langle \dddot{u}, i \rangle = \omega C \langle \dddot{u}, \dddot{u} \rangle = -\omega C U^2\] \hspace{1cm} (22)

In this case $\dot{P}$ and $\ddot{P}$ are negative defined quantities, while $P$ is zero. Under sinusoidal operation $\dot{P}$ and $\ddot{P}$ are equal and coincide with the capacitor reactive power.
4 Single-phase identification

In this section we describe a load identification procedure which, given actual load voltage $u$ and current $i$, identifies the simplest passive circuit which absorbs the same power terms $P$, $\hat{P}$ and $\tilde{P}$ of the actual load. With this procedure, the sign of the parameters of the passive circuit can be either positive or negative, according to the aim of the computation, i.e., load modeling or compensation. Moreover, a current/voltage orthogonal decomposition is introduced, which displays the effects of load and current distortion.

4.1 2nd-order identification

The 2nd-order identification of active and reactive current terms absorbed by a generic single-phase load under non-sinusoidal conditions, proposed in [1–4] and further discussed in [9, 12, 13], gives the values of the parameters of a parallel $R\!L\!C$ path, shown in Fig. 1.

![Fig. 1. Single-phase 2nd-order parallel identification](image)

which minimize the norm of residual current $i_r$ while keeping $i_v$ orthogonal to the other current components $i_u$ and $i_t$. In practice, let:

$$i_u = G u + i_c$$

$$i_t = \frac{A}{\omega} \tilde{u} + \omega C \tilde{u}$$

$$i_v = i - i_u - i_t$$

parameters $G$, $A$, $C$ are computed so as to minimize $\|i_r\|^2$. The result is:

$$G = \frac{P}{U^2}$$

$$A = \frac{\hat{P}}{U^2} \tilde{U}^2 - \tilde{P} U^2$$

$$\omega C = \frac{\hat{P}}{U^2} \tilde{U}^2 - \tilde{P} U^2$$

It is interesting to note that the same values of the parameters can be obtained by imposing the passive circuit of Fig. 1 to absorb total power terms $P$, $\hat{P}$ and $\tilde{P}$ entering terminals $A$ and $B$. Note also that, since $\tilde{U}^2 - \tilde{P} U^2 - U^4 \geq 0$, the sign of parameters $A$ and $C$ depends on the sign and amplitude of $\hat{P}$ and $\tilde{P}$.

Similarly, in the case of load identification by a series $R\!L\!C$ path, as shown in Fig. 2, we can determine the values of parameters $R$, $L$, $\Gamma$ which minimize $\|i_r\|^2$ by simply imposing that the passive circuit components absorb total power terms $P$, $\hat{P}$ and $\tilde{P}$ entering terminals $A$ and $B$. The result is:

$$R = \frac{P}{I^2}$$

$$\Gamma = \frac{\hat{P}}{I^2} \tilde{I}^2 - \tilde{P} \tilde{I}^2$$

$$\frac{\rho}{\omega} = \frac{\hat{P}}{I^2} \tilde{I}^2 - \tilde{P} \tilde{I}^2$$

Since $\tilde{I}^2 \tilde{I}^2 - I^4 \geq 0$, the sign of parameters $L$ and $\Gamma$ depends on sign and amplitude of $\hat{P}$ and $\tilde{P}$.

This 2nd-order identification is affected by two relevant limitations:

- The sign of the parameters can be either positive or negative, depending on the values of $\hat{P}$ and $\tilde{P}$. This makes impossible, in general, to perform positive-parameter identification for load modeling, or negative-parameter identification for load compensation. Only if $\hat{P}$ and $\tilde{P}$ have opposite sign the parallel (or series) representation offers the possibility to identify the load with both positive or negative reactive parameters, according to the goal of the identification. However, in most cases (e.g., when voltage and current distortion are limited) $\hat{P}$ and $\tilde{P}$ are nearly equal and have the same sign; thus, passive parameters can result with opposite sign.

- Under sinusoidal conditions eqs. (24) and (25) become undetermined. In fact, there is an infinite number of couples $A$ and $C$ which provides the same reactive power absorption of the actual load. Hereafter, we introduce a 3rd-order approach which removes both the above limitations while resulting in an orthogonal decomposition of the load current/voltage. We will consider separately the case of parallel identification and series identification. In all cases we will assume that the load absorbs a positive active power, so that it can be accounted for by a resistive cell. It is important to note that the proposed 3rd-order approach is developed using the following procedure: we first identify a single reactive element (either a capacitance or an inductance) so as to absorb one power-like term (either $\hat{P}$ or $\tilde{P}$, respectively) and then we identify the residual 2nd-order passive elements so as to absorb the residual power-like terms, as explained in more details in the next section. This approach, which imposes some constrains both in the structure and in the allowed passive parameters, allow us to directly control the sign of the passive bipoles and to give a solution even under sinusoidal conditions, thus removing the above-mentioned limitation of the 2nd-order approach.

As far as the resistive identification is concerned, either a parallel or a series resistance can be considered, as shown in Fig. 3. For the parallel resistive representation (Fig. 3a), conductance $G$ is selected so as to keep “active current” $i_a = G u$ orthogonal to “non-active cur-
current” $i_n = i - i_b$. Accordingly, the value of $G$ is $P/U^2$. This means that the conductance must absorb total active power $P$ entering the circuit. Similarly, for the series resistive identification (Fig. 3b), resistance $R$ is selected so as to keep “active voltage” $u_n = R \cdot i$ orthogonal to non-active voltage $u_n = u - u_n$. Accordingly, the value of $R$ is $P/U^2$ and the resistance $R$ absorbs total active power $P$.

Since conductance $G$ and resistance $R$ are not absorbing any other power term ($\hat{P}_a = 0$ and $\hat{P}_s = 0$), we can write:

\[
P_n = \langle u_n, i_n \rangle = 0 \Rightarrow \hat{P}_n = \langle u_n, \vec{i}_n \rangle = -\langle \vec{u}_n, i_n \rangle = \vec{P}
\]

\[
\hat{P}_n = \langle \vec{u}_n, i_n \rangle = -\langle u_n, \vec{i}_n \rangle = \vec{P}
\]

where we have indicated with $u_n$ and $i_n$ the output voltage and current of the ohmic cell. Of course, $u_n = u$ for the parallel representation and $i_b = i$ for the series representation. The circuit following the resistive cell includes a purely reactive section and a voltage or current generator, according to the general schemes of Fig. 4 (parallel identification) and Fig. 6 (series identification). The properties of the corresponding circuits are described thereafter.

**4.2 3rd-order parallel identification**

The 3rd-order equivalent parallel circuit which results from the load identification is shown in Fig. 4. It includes susceptance $B$, a 2nd-order reactive block $H$ (series or parallel $L-C$ connection), and either a residual current generator $i_d$ or a residual voltage generator $u_v$. The reactive elements are devised to absorb total power terms $\vec{P}$ and $\vec{P}$ entering the load, while residual generators do not absorb any power terms. In a first instance, we will search for an equivalent circuit with positive parameters (modeling approach).

**4.2.1 Case of $\hat{P} > 0$: Inductive susceptance**

Susceptance $B$ and the associated inductance $\frac{A_b}{\omega} = B$ are selected so as to keep current $i_d = i_n - i_b$ orthogonal to the remaining current $i_d = i_n - i_b$.

Thus:

\[
\langle i_b, i_d \rangle = 0 \Rightarrow \langle i_b, i_n - i_b \rangle = \langle B \cdot \vec{u}_n, i_n - i_b \rangle = B(\vec{P} - \hat{P}_b) = 0 \Rightarrow \hat{P} = \hat{P}_b = B \cdot \vec{U}_n^2
\]

The resulting value of the susceptance is given by:

\[
B = \frac{\Lambda_b}{\omega} = \frac{\vec{P}}{\vec{U}_n^2}
\]

and is positive if $\hat{P} > 0$, which is the case under consideration. Note that susceptance $B$ does not absorb active power ($\hat{P}_b = 0$), while it absorbs the entire $\hat{P}$, so that term $\hat{P}_d = \langle \vec{u}_n, i_d \rangle$ conveyed by current $i_d$ is zero. Instead, term $\hat{P}_d = -\langle \vec{u}_n, i_d \rangle$ is generally different from zero. It is given by:

\[
\hat{P}_d = \vec{P} - \hat{P}_b = \vec{P} - \frac{\Lambda_b}{\omega} \cdot \vec{U}_n^2 = \vec{P} - \hat{P} \cdot \frac{\vec{U}_n^2}{\vec{U}_n^2} = -\frac{\hat{P} \cdot \vec{U}_n^2 - \vec{P} \cdot \vec{U}_n^2}{\vec{U}_n^2}
\]

Note that $\hat{P}_d$ is zero if voltage $u_n$ is purely sinusoidal (in this case, in fact, $\hat{P} = \vec{P}$ and $\vec{U}_n = u_n$). In this instance, the following block $H$, whose function is to absorb the remaining power term $\hat{P}_d$, is not needed; moreover, voltage source $u_v$ is zero and residual current $i_v$ coincides with $i_d$.

Instead, in presence of supply voltage distortion, the 2nd-order reactive block $H$ has the function to absorb $\hat{P}_b = \hat{P}_b$ while keeping $\hat{P}_b = \hat{P}_b = 0$. These conditions can be met by a purely reactive 2nd-order circuit. The identification can be done according to equations (24) and (25), by substituting $\hat{P}_d$ in place of $\hat{P}$ and zero in place of $\hat{P}$. The result is:

- for the parallel representation:

\[
\frac{\Lambda_b}{\omega} = -\frac{\hat{P}_d \cdot \vec{U}_n^2}{\vec{U}_n^2 \cdot \vec{U}_n^2 - \vec{U}_n^2} = -\frac{\vec{P}_d \cdot \vec{U}_n^2}{\vec{U}_n^2 \cdot \vec{U}_n^2 - \vec{U}_n^2}
\]

- for the series representation:

\[
\omega \cdot C_h = -\frac{\hat{P}_d \cdot I_d^2}{I_d^2 \cdot I_n^2 - I_d^2} = \frac{\hat{P}_d \cdot I_d^2}{I_d^2 \cdot I_n^2 - I_d^2}
\]

Note that, if we search for positive parameters, the parallel representation applies for $\hat{P}_d < 0$, while the series representation applies for $\hat{P}_d > 0$. Correspondingly, we refer to the two equivalent circuits of Fig. 5a and 5b.

In the case of $\hat{P}_d < 0$, the equivalent circuit is that of Fig. 5a, the parameters being given by eq. (28). The resulting susceptances $\frac{\Lambda}{\omega} = \frac{\Lambda_b + \hat{A}_b}{\omega}$ and $\omega \cdot C_h$ co-
incide with those given by eq. (24). This shows that the
2nd-order representation of Fig. 1 is a special case of
the 3rd-order representation and applies whenever \( \tilde{P}_d < 0 \), i.e. \( \tilde{P} \leq U_0^2 \) (including the case of \( \tilde{P}_d < 0 \)).

Total reactive current \( i_r = i_b + i_h \) is orthogonal to
residual current \( i_v \), so that we can write:

\[
\hat{I}^2 = I_A^2 + I_B^2 + I_D^2 = I_A^2 + I_i^2 + I_v^2
\]

(30)

In the case of \( \tilde{P}_d > 0 \), i.e. \( \tilde{P} \leq U_0^2 \), we apply
eq (29) and refer to the equivalent circuit of Fig. 5b,
where voltage generator \( u_n \) does not absorb power
terms (\( P_d = \tilde{P} = \tilde{V} = 0 \)) is orthogonal to voltage \( u_v \),
and has minimum norm. Parameters \( L_h \) and \( C_h \) become
underdefined if current \( i_d \) is purely sinusoidal, which
however corresponds to a very strange situation, once
we assume that the supply voltage is distorted.

4.2.2 Case of \( \tilde{P} < 0 \): Capacitive susceptance

In this case, applying the same procedure shown
before, we obtain

\[
B = \omega \ C_h = -\frac{\tilde{P}}{U_n^2}
\]

(31)

which is positive if \( \tilde{P} < 0 \). In presence of supply voltage
distortion the 2nd-order reactive block \( H \) must absorb
\( P_h = \tilde{P} = -i_h \) while keeping \( P_b = \tilde{P} = 0 \). Also in this case, the identification can be done according to
\( (24) \) and \( (25) \), by substituting \( P_d \) in place of \( \tilde{P} \) and
zero in place of \( P \). Thus, if we search for positive parameters, the parallel representation applies for \( \tilde{P}_d > 0 \),
while the series representation applies for \( \tilde{P}_d < 0 \). The equivalent circuits are obtained from those of Fig. 5a
and 5b by substituting capacitive \( C_h \) in place of iner-
tance \( A_h \). The properties are those discussed in Section
4.2.2.

4.3 3rd-order series identification

The 3rd-order equivalent series circuit is shown in
Fig. 6. It includes reactance \( X \), a 2nd-order reactive
block \( H \) (series or parallel \( L-C \) connection), and either
a residual current generator \( i_M \) or a residual voltage
generator \( u_n \). The reactive blocks are devised to absorb
total power terms \( \tilde{P} \) and \( \tilde{P} \) entering the load, while re-
sidual generators do not absorb any power terms. We
will search for an equivalent circuit with positive pa-
rameters (modeling approach).

4.3.1 Case of \( \tilde{P} > 0 \): Inductive reactance

Reactance \( X \) and the associated inductance \( L_h = X / \omega \)
are selected so as to keep voltage \( u_n \) orthogonal to the
remaining voltage \( u_d = u_n - u_h \). Thus:

\[
X = \omega \ L_h = \frac{\tilde{P}}{I_n^2}
\]

(32)

Since reactance \( X \) absorbs the entire \( \tilde{P} \) term \( \tilde{P}_d = \langle u_d, i_d \rangle \) conveyed by voltage \( u_d \) is zero. Instead, term
\( \tilde{P}_d = -\langle u_d, i_n \rangle \) is generally different from zero under
non-Sinusoidal conditions. The 2nd-order reactive block
\( H \) has the function to absorb \( \tilde{P}_h = \tilde{P}_d \) while keep-
ing \( \tilde{P}_b = \tilde{P}_b = 0 \). These conditions can be met by a
purely reactive 2nd-order circuit. The identification is
done again according to eqs. \( (24) \) and \( (25) \), by substi-
tuting \( \tilde{P}_d \) in place of \( \tilde{P} \) and zero in place of \( \tilde{P} \). Thus, if we
search for positive parameters, the parallel represen-
tation applies for \( \tilde{P}_d > 0 \), while the series representa-
tion applies for \( \tilde{P}_d < 0 \). The corresponding equivalent circuits are immediately derived.

4.3.2 Case of \( \tilde{P} < 0 \): Capacitive reactance

In this case, applying the same procedure shown
before, we obtain

\[
X = \frac{I_h}{\omega} = -\frac{\tilde{P}}{I_n^2}
\]

(33)

Since reactance \( X \) absorbs the entire \( \tilde{P} \) term \( \tilde{P}_d = -\langle u_d, i_n \rangle \) conveyed by voltage \( u_d \) is zero. Instead, term
\( \tilde{P}_d = \langle u_d, i_n \rangle \) is generally different from zero under
non-Sinusoidal conditions. The 2nd-order reactive block
\( H \) has the function to absorb \( \tilde{P}_h = \tilde{P}_d \) while keeping \( \tilde{P}_b = \tilde{P}_b = 0 \). These conditions can be met by a purely
reactive 2nd-order circuit. The identification is done
again according to eqs. \( (24) \) and \( (25) \), by substi-
tuting \( \tilde{P}_d \) in place of \( \tilde{P} \) and zero in place of \( \tilde{P} \). Again, if we search for positive parameters, the parallel representation applies for \( \tilde{P}_d < 0 \), while the series representation applies for \( \tilde{P}_d > 0 \).
4.4 3rd-order identification for load compensation

If the identification is oriented to load compensation, it makes sense to search for equivalent circuit components with negative parameters (besides the resistive term, of course). This allows an easy design of the compensation network, which keeps the same structure of the identified load but includes positive parameters, opposite to those of the identified load.

Thus, for any given \( P \) and \( P' \) we have two possible equivalent circuits. Taking into account that we also connect a series or parallel resistive cell to account for the active power absorption, it results that for any load voltage \( u \) and current \( i \) there are generally four possible circuit configurations suitable to identify the load. In all cases, the residual voltage or current generator represents the action that cannot be performed by the 3rd-order compensation circuit and thus requires an active series or parallel active filter for the compensation.

5 Application example

As an example of application, let us consider a load which includes a thyristor rectifier (0.7 pu, \( \alpha \sim 35^\circ \)) and a linear ohmic-inductive load (0.2 pu, \( \cos \phi = 0.7 \)). Load voltage \( u \) (phase \( a \)) and current \( i \) (phase \( a \)) are reported in Fig. 7a and 7b, together with the corresponding active \( u_a, i_a \) and non-active terms (\( u_b, i_b \)). We consider only voltage and current of phase \( a \) in the assumption that the system is balanced.

For voltage \( u \) and current \( i \) of Fig. 7, the above identification method brings to the four circuits (with negative reactive terms) shown in Fig. 8. Figs. 8a and 8b refer to a parallel hybrid compensator and Figs. 8c and 8d to a series one. The values obtained with the proposed procedure are shown in Tab. 1, where the apparent power of the residual generator (\( S_{\text{fa}} \)), divided by the apparent power of the load (\( S \)), is also reported. This parameter shows that, among the four possible solutions, the scheme of Fig. 8b minimizes the active filter rating. It is worth noting that the solution of Fig. 8b resembles the hybrid active filter discussed in [14], where a single \( L-C \) cell is used in series with an active filter to provide a parallel load compensation.

![Fig. 7](image1.png)

**Fig. 7.** (a) line voltage \( u \) and its decomposition into active (\( u_a \)) and non-active (\( u_b \)) terms; (b) load current \( i \), and its decomposition into active (\( i_a \)) and non-active (\( i_b \)) terms.

![Fig. 8](image2.png)

**Fig. 8.** Possible topologies of proposed 3rd-order identification using load current \( i \) and voltage \( u \) of Fig. 7.

<table>
<thead>
<tr>
<th>G-R [pu]</th>
<th>B-X</th>
<th>( L_h-C_h ) [pu]</th>
<th>( S_{\text{fa}}/S_{\text{load}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( G = 0.676 ) B = –0.549 ( \omega L_h = –0.25 ) ( 1/(\omega C_h) = –27.2 ) 0.097</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) ( G = 0.676 ) ( 1/X + \omega C_h = –0.598 ) ( \omega L_h = –0.018 ) 0.022</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) ( R = 0.816 ) B = –0.317 ( \omega L_h = –0.022 ) ( 1/\omega C_h = –0.957 ) 0.150</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d) ( R = 0.816 ) ( 1/X + \omega C_h = –1.369 ) ( \omega L_h = –0.0002 ) 0.159</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Tab. 1.** Circuit parameters of Fig. 8

In order to illustrate the properties of the proposed decomposition, Figs. 9–12 show the current and voltage decomposition using the schemes of Figs. 8a–d. In Fig. 9 note that capacitive current \( i_b \) includes most of the fundamental reactive component;
moreover, it shows non-negligible distortion, mainly due to the harmonics in the supply voltage. In Fig. 10, instead, capacitive voltage \( u_b \) nearly coincides with supply voltage \( u \), so that the remaining voltage on cell \( L_h - C_h \), as well as residual voltage \( u_v \), are very small. As far as the series compensation is concerned, Fig. 11 shows that, with the solution of Fig. 8c, the high-level of distortion in the non-active voltage \( u_n \) is amplified by capacitor \( C_b \), so that the second-order cell is fed by a distorted non-active voltage \( u_n \) and a highly distorted current \( i_d \). Finally, in Fig. 12 we note that most of the fundamental component of load voltage \( u \) is included in voltage \( u_b \), while the second-order cell absorbs most of the voltage distortion of \( u_n \), which drops mainly on residual voltage generator \( u_v \). It is interesting to note that, even if parameter \( C_h \) and \( L_h \) are always chosen so as to minimize the rms value of the residual voltage generator, the results of the minimization process can be quite different due to the previous choice for the configuration of the resistive and fundamental reactive cell. Thus, in order to find the optimal solution within the allowed passive structures, all the possible four configurations need to be analyzed, as performed in this example.

6 Conclusions

Based on suitable power-like terms definition, the paper discussed an extension of the 2nd-order load identification, proposed in the literature, into a 3rd-order identification method, which removes the main limitations of the previous approach and results in a practical tool for load modeling and design of compensation systems based on hybrid filters.

7 List of symbols

\[
\begin{align*}
\left\langle \right\rangle & \quad \text{scalar product in } L^2 \\
\| \| & \quad \text{norm in } L^2 \\
\omega & \quad \text{fundamental angular frequency} \\
u, \bar{u}, \tilde{u} & \quad \text{line voltage, its “derivative” and “integral” terms} \\
i, \bar{i}, \tilde{i} & \quad \text{load current, its “derivative” and “integral” terms} \\
U, \bar{U}, \tilde{U} & \quad \text{rms value of line voltage, its “derivative” and “integral” terms} \\
I, \bar{I}, \tilde{I} & \quad \text{rms value of load current, its “derivative” and “integral” terms} \\
p, \bar{P}, \tilde{P} & \quad \text{active power, derivative and integral power term} \\
G, R & \quad \text{equivalent conductance (resistance)} \\
L, A, C, \Gamma & \quad \text{equivalent reactive components for 2nd-order identification} \\
L_h, \Lambda_h, C_h, \Gamma_h & \quad \text{equivalent passive components for blocks } B \text{ and } X \\
L_h, \Lambda_h, C_h, \Gamma_h & \quad \text{equivalent passive components for the second order block } H \\
i_a (i_n) & \quad \text{active (non-active) currents} \\
u_a (u_n) & \quad \text{active (non-active) voltages} \\
i_b (u_b) & \quad \text{current (voltage) on reactive block } B \text{ (X)} \\
i_d (u_d) & \quad \text{residual current (voltage) on the second order block } H \text{ for parallel (series) identification}
\end{align*}
\]
References


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