Reactive Power and Unbalance Compensation Using STATCOM with Dissipativity-Based Control
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Abstract—The paper investigates the use of dissipativity-based control for voltage-sourced inverters (VSI) utilized as reactive power and unbalance compensators, e.g., Static synchronous compensator (STATCOM). The approach relies on a frequency domain modeling of system dynamics using positive sequence and negative sequence dynamic components. Following the passivity-based procedure, a family of controllers is obtained whose objective is to deal with unbalanced supply voltages, and to perform regulation of ac unbalanced currents. The proposed solution is relevant for high-power VSI inverters where, due to switching frequency limitations, conventional regulators do not have enough bandwidth to achieve reference tracking. New controllers include adaptive refinements to cope with parameter uncertainties, and stability and convergence proofs are outlined in all cases. Finally, a series of simplifications and adjustments have been explored to reduce the adaptive controllers to PI structures in stationary frame coordinates. The simplified controllers are in a form suitable for implementation, and establish a downward compatibility with standard industrial practice.

Index Terms—Active filters, adaptive control, dissipative systems, nonlinear systems, reactive power control.

I. INTRODUCTION

Reactive power compensators based on voltage-sourced inverters (VSI), such as Static synchronous compensator (STATCOM) [6], are finding increasing applications at different power levels. They offer several advantages over conventional thyristor-based converters solutions in terms of the speed of response, flicker compensation, flexibility, and minimal interaction with the supply grid.

A high-performance VSI control of both ac currents and dc-link voltage is not a straightforward task, especially during unbalanced operation. For VSIs switching at low frequency (or based on multilevel converters) the stationary reference frame regulators usually do not achieve acceptable performance because of bandwidth limitations. For these applications, usually at high power levels, synchronous frame (dq) regulators are required in practice to achieve satisfactory reference tracking. Numerous control strategies have been proposed for this purpose in recent publications [1], [2], [5], [7], [10]. The most widely used are PI controllers [2], [4], PI controllers with decoupling [3], and robust controllers (H_2 and H_{\infty}) [4]. Besides the requirement of decoupling between the current and dc voltage loops and of small-signal approximation in spite of nonlinear dynamic behavior, most of the proposed solutions are unable to deal with unbalanced operating conditions in supply voltages or in loads.

In this paper we investigate the use of a dissipativity theory for the dynamic control of VSI utilized either as a STATCOM for reactive and unbalance compensation, or for high-quality rectification. The two applications are treated here in a unified framework, as they have similar dynamic behavior. We use a frequency domain description of system dynamics based on positive sequence and negative sequence dynamic components to deal with unbalance in the supply voltages or in the load currents.

Following the passivity-based design procedure [8], a family of controllers aimed at achieving the ac current and dc-link voltage regulation is obtained by controlling the suitably defined energy of the closed-loop system, and adding the required damping. Refinements of the passivity-based procedure are made in terms of adaptation which allows to cope with the unavoidable parameter uncertainties. The first controller obtained via this procedure features a dynamical damping term and is denoted as controller A. We also derive a simplified version that we denote as controller B. Next, we use a decoupling assumption involving the dc-link voltage and ac currents to show how a similar controller (controller C) can be obtained from different considerations. In every case, we outline convergence and stability proofs. Moreover, we show that after some simplifications and adjustments, the proposed adaptive controllers reduce to PI structures with feedforward. To facilitate the implementation, we follow the ideas of [13] to express the controllers in terms of the stationary frame quantities. Simulations are provided to assess the performance of the controller B (the most relevant for practice) in comparison with a conventional proportional integral (PI) controller.

II. SYSTEM MODEL

A three-phase three-wire of VSI working either as STATCOM or as PWM rectifier is depicted in Fig. 1. The dynamics of this system can be expressed by the following model (see, e.g., [9] for further details about the modeling process):

\[ v(t) = v(t) - v_{\text{ref}}(t) = -L \frac{d}{dt} i(t) - r i(t) + v_S(t) \]
\[ C \frac{d}{dt} v_C(t) = \frac{(\delta(t))^2 v_C(t)}{2} - \frac{v_C(t)}{R_L} \]
where
\[ I(t) \] compensating currents \( \in \mathbb{R}^3; \)
\[ I_L(t) \] currents produced by the load \( \in \mathbb{R}^3; \)
\[ v_S(t) \] voltages coming from the source (referred to “n”) \( \in \mathbb{R}^3; \)
\[ v(t) \] voltages at the input of the VSI (referred to “n”) \( \in \mathbb{R}^3; \)
\[ v'(t) \] \( = \frac{v_S(t) \delta(t)}{2} \) voltages at the input of the VSI (referred to VSI neutral point “o”) \( \in \mathbb{R}^3; \)
\[ v_{no}(t) \] voltage between point “n” and “o” \( \in \mathbb{R}; \)
\[ \delta(t) \] vector of switch positions \( \in \{-1,1\}^3; \)
\[ L \] filter inductance (same for each line);
\[ r \] parasitic resistance of filter inductance (same for each line);
\[ C \] output capacitor;
\[ R_L \] resistive element collecting switching (and other) losses plus an optional load resistance.

The second equation is derived from the power balance across the lossless VSI. Due to the fact that \( i_1(t) + i_2(t) + i_3(t) = 0 \), taking parameters \( L \) and \( r \) same for each branch and assuming that \( v_{S1}(t) + v_{S2}(t) + v_{S3}(t) = 0 \), we obtain

\[ v_1(t) + v_2(t) + v_3(t) = 0 \]
\[ v_{no}(t) = \frac{v_1(t) + v_2(t) + v_3(t)}{3} \] (1)

out of which the vector of voltages at the input of the VSI (referred to the neutral “n” of the power grid) can be computed as follows:

\[ v(t) = v'(t) - v_{no}(t) = \frac{v_S(t) \delta(t)}{2} \]
\[ B = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}. \]

For the control design purpose we will consider the averaged variant of the model described above. We assume that a sufficiently high frequency is used to implement the switching control sequence (using for instance pulse width modulation), so that we can replace vector \( \delta \) by the corresponding switching voltage \( u = [u_1, u_2, u_3]^T \) which is a vector of continuous signals approximating the actual controller.

Under these considerations the averaged model can be written as follows:

\[ L \frac{d}{dt} i(t) = -r i(t) - \frac{v_C(t)}{2} B u(t) + v_S(t) \] (2)
\[ C \frac{d}{dt} v_C(t) = \frac{u(t)^T i(t)}{2} - \frac{v_C^2(t)}{R_L}. \] (3)

Notice that the term \( v_C u \), appearing in the system model, will generate harmonics other than those associated with each factor, unless the control \( u \) contains a term that directly cancels \( v_C \) (as done implicitly by the transformation proposed later). A way to avoid this problem is to assure that \( u \) cancels harmonics of \( v_C \); this will be achieved implicitly by the input transformation that we consider next. In addition to rejecting the wanted harmonics we aim to linearize the dynamics of the current subsystem, and to “decouple” the currents with respect to the dynamics of \( v_C \).

As shown in Fig. 1, the power converter directly feeds an equivalent resistance \( R_L \) on the dc side, which represents the load for rectifier applications. In case of only reactive power and unbalance compensation, the load is not present, and resistance \( R_L \) accounts only for VSI losses. Therefore, this setup allows us to describe in unified manner the case in which the VSI converter works as a rectifier and the case in which the same converter works as a STATCOM. For that purpose, line currents \( i_S \) can be expressed as

\[ i_S = i + \sigma i_L \] (6)

where \( \sigma = 1 \) for STATCOM application and \( \sigma = 0 \) for PWM rectifiers.

The system is of course affected by uncertainties in various parameters, such as \( L, C \), and resistive parameters \( r \) and \( R_L \), whose values are considered constant (but unknown), or slowly varying (including step changes); this makes the control design much more challenging.

### A. Dynamic Phasor Model

Following the standard notation, we introduce \( \alpha = e^{j \omega_T/3} \), then the square of \( \alpha \) equals the complex conjugate \( \alpha^2 = \bar{\alpha} \). Then for \( \tau \in [t - T; t] \) and \( w = 2\pi fT \) a three-phase time-domain waveform can be written as

\[ \begin{pmatrix} x_O \\ x_P \\ x_L \end{pmatrix}(\tau) = \sum_{\omega = -\infty}^{\infty} e^{j \omega \tau} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ \bar{\alpha} & \alpha & 1 \\ \bar{\alpha} & \alpha & 1 \end{bmatrix} \begin{bmatrix} X_O^P \\ X_P^O \\ X_L^O \end{bmatrix}(t) \] (7)
and we denote the square transformation matrix with $A$. It can be checked that $A$ is unitary, as $A^{-1} = A^H$, where $H$ denotes complex conjugate transpose (Hermitian). As commonly encountered in transforms, scaling factors other than $1/\sqrt{3}$ are possible in the definition of matrix $A$, but they require adjustments in the inverse transform. The coefficients in (7) are

$$
\begin{bmatrix}
X_P^d \\
X_P^o \\
X_P^i
\end{bmatrix}(t) = \frac{1}{T} \int_{t-T}^{t} e^{-j\omega \tau} A^H \begin{bmatrix}
x_a \\
x_b \\
x_c
\end{bmatrix}(\tau) d\tau.
$$

The equation (8) defines *dynamical* positive $X_P^d$, negative $X_P^o$, and zero-sequence $X_P^i$ symmetric components at frequency $\omega$, as

$$
\frac{d}{dt} \begin{bmatrix}
X_P^d \\
X_P^o \\
X_P^i
\end{bmatrix}(t) = A^H \begin{bmatrix}
\left(\frac{dX_P^d}{dt}\right)_\ell \\
\left(\frac{dX_P^o}{dt}\right)_\ell \\
\left(\frac{dX_P^i}{dt}\right)_\ell
\end{bmatrix}(t) - j\omega \begin{bmatrix}
X_P^d \\
X_P^o \\
X_P^i
\end{bmatrix}(t)
$$

where $\langle \cdot \rangle_\ell$ is defined at time $t$ by the following averaging operation:

$$
\langle x(t) \rangle_\ell(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau)e^{-j\omega \tau} d\tau.
$$

Because of our focus on PWM rectifiers and/or reactive power and unbalance compensators, we will consider only the fundamental or main harmonic, so $\ell = \pm 1$. Moreover, we found it more convenient to work with real vectors in the place of complex numbers for the purpose of control design; thus in our derivations we split all signals in real and imaginary parts, and consider them as real vectors. For instance, $X_P^d = [X_P^d, X_P^o, X_P^i]$, where $X_P^d = \Re\{X_P^d\}$ and $X_P^o = \Im\{X_P^o\}$.

Using (9) to system (4)–(5) for $\ell = \pm 1$, we obtain a set of dynamic phasor models (each phasor being represented as real vector)

$$
\begin{align*}
\frac{d}{dt} L_{dc} I_{dc} &= -wL_{dc} I_{dc} - r I_{dc} - E_{dc} V_{dc} + V_{Sdc}, \\
C \frac{d}{dt} \dot{\theta} &= \sum_{\ell=-1,\ell\neq0}^{1} E_{dc,\ell} I_{dc,\ell} - 2\dot{\theta},
\end{align*}
$$

where we have defined $\theta \triangleq (1/R_L) \cdot z = (\xi_0^2/2)\theta_0$, $\xi_0$ stands for the dc component; we have used the fact that $ABA^{-1} = I_4$ with $I_4$ an identity matrix; $z = -2^T$ is a block diagonal matrix of skew symmetric matrices

$$
\mathcal{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.
$$

Matrix $\mathcal{J}$ has been used in the place of the imaginary number $j$ to express all variables as real vectors while preserving the properties of all operations.

Using the fact that $\langle (t)^T \dot{z}(t) \rangle_0 = \sum_{\ell=-1,\ell\neq0}^{1} E_{dc,\ell} I_{dc,\ell} = 2E_{dc,\ell=1} I_{dc,\ell=1}$, and since the structure of subsystem (11) is the same for either $\ell = 1$ or $\ell = -1$, we can consider only the case $\ell = 1$ in (11), and the results will apply mutatis mutandi for $\ell = -1$. This yields (eliminating index $\ell$)

$$
\begin{align*}
\frac{d}{dt} I_{dq} &= -wL_{dc} I_{dq} - r I_{dq} - E_{dq} V_{Sdq}, \\
C \frac{d}{dt} \dot{\theta} &= E_{dq}^T I_{dq} - \dot{\theta},
\end{align*}
$$

Under these conditions, vectors $I_{dq}$, $E_{dq}$, $V_{Sdq}$ have four components, for instance $I_{dq} = [I_{dq}^d, I_{dq}^o, I_{dq}^i]^T$. Note that the zero-sequence symmetric component is not taken into account since the considered three-phase system is without the neutral conductor ("three wire" system). Indeed, since $i_1 + i_2 + i_3 = 0$, the zero-sequence current is always zero, and any possible zero-sequence component in the voltages (such as in $v(t)$) is not changing the dynamic behavior of our systems and can be neglected; this is typical in the so called $dq$–control of converters.

The transformation shown above includes a rotation by $\pm\omega t$, so the corresponding model is denoted as $dq$–$PN$ model, with the inherent advantages of dealing with variables converging toward constant references. This is an important point especially for high-power low-switching frequency VSI or multilevel VSI [12], where stationary reference frame regulators do not have enough bandwidth to provide acceptable reference tracking.

### B. Control Objective in Terms of Phasor $(dq–pm)$ Variables

Different control objectives can be formulated depending on the particular application; one widely used goal is to control the line currents to be proportional to line voltages, to achieve a unity power factor from the line side. In dynamic phasor modeling, this means that the same constant of proportionality is used both for the positive sequence and negative sequence components, i.e.,

$$
i_S = g_T i_S \iff \begin{bmatrix} I_{Sdq}^d \\
I_{Sdq}^o \\
I_{Sdq}^i
\end{bmatrix} = \begin{bmatrix} g_T I_{pdq}^d \\
g_T I_{pdq}^o \\
g_T I_{pdq}^i
\end{bmatrix}
$$

where $g_T$ is representing the apparent conductance observed by the power source giving a measure of the total active power absorbed from the supply line, which can be theoretically split between a term $g_T$ representing the power absorbed by the load and a term $g$ representing VSI losses, i.e., $g_T = g_T' + g$.

Evaluation of term $g_T$ is performed by a VSI controller. Recalling that $I_{Sdq} = \sigma I_{Ldq} + I_{dq}$ ($\sigma = 1$ for STATCOM and $\sigma = 0$ for PWM rectifier), then the reference for $I_{dq}$ directly comes from the reference of line currents and possibly from the load current measurements, that is,

$$
I_{pdq}^d = I_{Sdq}^d - I_{Ldq} = g_T V_{Sdq} - I_{Ldq}.
$$

The problem is then translated into a tracking problem on the VSI currents $I_{dq}$, where the computation of the desired reference currents $I_{pdq}^d$ constitutes one of the main issues in control design. We assume that fundamental components $V_{Sdq}$ and $I_{Ldq}$ are constant or slowly varying vectors (including possible step changes).

The goal stated above can only be accomplished if enough energy is stored into the dc-link capacitor so that it is able to inject the desired currents $I_{pdq}^d$; this task is accomplished by regulating the output capacitor voltage $v_C$ toward a desired constant value. This objective is the most important one when the system is operated as a PWM rectifier.

### III. ADAPTIVE PBC STRATEGY

Following the passivity-based control (PBC) method [8], at the first step we reshape the energy of the system, that is, we pro-
pose to modify the energy of the system to reach a desired equilibrium state. The new energy function will preserve the same structure as the original one, but its minimum will be located at the desired operating point. The original energy function of the system is given by

\[ W_0 = \frac{C}{4} z^2 + \frac{L}{2} |I_{dq}|^2 \]

and the new energy function is given by

\[ W_1 = \frac{C}{4} \hat{z}^2 + \frac{L}{2} |\hat{I}_{dq}|^2 \]  \hspace{1cm} (15)

where \( |\cdot| \) stands for the module of a vector and we have defined \( \hat{I}_{dq} \triangleq (I_{dq} - Y) \) and \( \hat{z} \triangleq (z - \zeta) \), with \( \zeta \) and \( Y \) the desired auxiliary states.

The quantity \( W_1 \) represents the energy of a system of the form

\[ \dot{L}_{\hat{d}} = -(r + wLz)\hat{I}_{d} + \hat{L} \left( w\hat{Y} + \frac{d}{dt}\hat{Y} \right) + \hat{r}Y + \phi_1 \]
\[ \frac{C}{2} \hat{z} = -\theta \hat{z} + \hat{\theta} \zeta + \phi_2 \]  \hspace{1cm} (16) \hspace{1cm} (17)

which can be obtained from the original system by adding and subtracting the signals \( \phi_1 \) and \( \phi_2 \) defined as

\[ \phi_1 \triangleq -\hat{L} \frac{d}{dt} \hat{Y} - E_{dq} + V_{sdq} - \hat{r}Y - w\hat{L}\hat{Y} \]
\[ \phi_2 \triangleq -\frac{C}{2} \frac{d}{dt} \hat{z} + \hat{E}_{dq} - \hat{\theta} \zeta \]

where \( \hat{\cdot} \) represents the estimates of the unknown parameters, and \( \hat{\cdot} \triangleq \frac{\cdot}{\cdot} \).

The second step in PBC design, referred to as damping injection, is aimed at incorporating dissipation into the system, and thus increasing the likelihood of system stability. To add the required damping into the system (16)–(17) we select

\[ \phi_1 = -K_1 \hat{I}_{d} \] \hspace{1cm} (20)
\[ \phi_2 = -K_2 \zeta, \quad \hat{\eta} = -b\eta + a\hat{z} \] \hspace{1cm} (21)

which in this case includes a dynamic extension. The idea behind this “dynamical” damping is, on the one hand, to prevent the introduction of more harmonics into the control system by filtering \( \hat{z} \) before feedback, and, on the other hand, to enable access to the time derivative of \( g_T \) in the implementation of the controller and adaptive laws, as will be explained later. This yields the following error model:

\[ \dot{L}_{\hat{d}} = -(r + K_1 + wLz)\hat{I}_{d} + \hat{L} \left( w\hat{Y} + \frac{d}{dt}\hat{Y} \right) + \hat{r}Y + \phi_1 \]
\[ \frac{C}{2} \hat{z} = -\theta \hat{z} + \hat{\theta} \zeta - k_\psi \eta \]
\[ \dot{\hat{\eta}} = -b\eta + a\hat{z} \]  \hspace{1cm} (22) \hspace{1cm} (23) \hspace{1cm} (24)

At the third step, we solve for the control signal \( E_{dq} \) from (18)–(19), with \( \phi_1, \phi_2 \) as chosen in (20)–(21), and restricting the auxiliary states to \( \hat{Y} = I_{dq}^* \) and \( \zeta = z^* \), where \( I_{dq}^* \) is computed as in (14) and \( z^* = V_{d}^2 / 2 \). This yields the following expressions for the controller:

\[ E_{dq} = V_{sdq} - \hat{r}I_{dq} - \hat{L} \left( w\hat{Z}I_{dq}^* + \frac{d}{dt}I_{dq}^* \right) + K_1 \hat{I}_{dq} \]  \hspace{1cm} (25)
\[ 0 = E_{dq}^2 \hat{I}_{dq} - \hat{\theta} z^* - k_\psi \eta \]  \hspace{1cm} (26)

Due to the introduction of the dynamics (24), the new energy function is now extended to \( W_2 = W_1 + (k_\psi / 2\alpha) \eta^2 \) whose time derivative along the trajectories of (22)–(23) is given by

\[ \dot{W}_2 = -(r + K_1) \hat{L}_{dq} - \theta \hat{z}^2 + \hat{r} \hat{I}_{dq}^* - \hat{L} \left( w\hat{Z}I_{dq}^* + \frac{d}{dt}I_{dq}^* \right) - \hat{\theta} z^* \hat{z} - \frac{k_\psi \eta^2}{\alpha} \]

To deal with the terms associated with the error signals \( \hat{r}, \hat{L} \), at the fourth and last step we complete (15) as \( W = W_2 + (1/2\gamma_2) \hat{L}^2 + (1/2\gamma_2) \hat{r}^2 \), whose time derivative is forced to be negative semidefinite if we propose to construct the error signals according to the following adaptive laws:

\[ \dot{\hat{r}} = -\gamma_1 \hat{I}_{dq}^* \]
\[ \dot{\hat{L}} = -\gamma_2 \hat{I}_{dq}^* \left( w\hat{Z}I_{dq}^* + \frac{d}{dt}I_{dq}^* \right) \]
\[ \dot{\theta} = -\gamma_3 \hat{z}^* \hat{z} \]  \hspace{1cm} (27) \hspace{1cm} (28) \hspace{1cm} (29)

where \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) are positive design parameters. This yields

\[ W = -(r + K_1) \hat{L}_{dq}^2 - \theta \hat{z}^2 - \frac{k_\psi \eta^2}{\alpha} \]

From LaSalle’s theorem, \( W \equiv 0 \) for \( \hat{L}_{dq} = \hat{z} = \eta = 0 \), which implies that \( \dot{\hat{r}} = \dot{\hat{L}} = \dot{\theta} = 0 \) and finally \( \hat{r} = \hat{L} = 0 \). Hence, \( \hat{L}_{dq}, \hat{z} \) and \( \eta \) approach zero asymptotically, guaranteeing convergence of parameters \( \hat{r}, \hat{L} \) toward their true values.

To guarantee that all internal signals are bounded, we should still establish boundedness of \( I_{dq}^* \) and its time derivative. This will be shown (in an approximate way) next.

Developing expression (26), with \( E_{dq} \) defined as in (25), we obtain

\[ V_{sdq} I_{dq}^* = \hat{r} |I_{dq}^*|^2 + \hat{L} (I_{dq}^*)^T \frac{d}{dt} I_{dq}^* + \hat{\theta} z^* - k_\psi \eta - \epsilon(t) \]  \hspace{1cm} (30)

where we have defined the decaying signal \( \epsilon(t) = [K_1 I_{dq} + V_{sdq} \hat{L} (w\hat{Z}I_{dq}^* + \frac{d}{dt}I_{dq}^*) + \hat{r} I_{dq}]^T \hat{I}_{dq} \). Recalling that \( I_{dq}^* = g_T V_{sdq} - I_{dq} \) we get

\[ V_{sdq} I_{dq}^* = g_T |V_{sdq}|^2 - V_{sdq} \hat{L}_{dq}, \quad \frac{d}{dt} I_{dq}^* = g_T V_{sdq} \]

Direct substitution in (30) yields

\[ g_T |V_{sdq}|^2 = \hat{\theta} z^* + V_{sdq} \hat{L}_{dq} + k_\psi \eta + \epsilon(t) \]  \hspace{1cm} (31)

From this expression we can solve for \( g_T \) which is required to compute the time derivative of \( I_{dq}^* \) used in controller (25) and adaptive law (28), and then solve it online to get \( g_T \) used in \( I_{dq}^* \). Evidently, this will result in a fairly complicated controller that may be difficult to implement. Nevertheless, the complexity of this part of the controller can be reduced considerably under certain physically meaningful assumptions. First, since \( r \) and \( L \) are usually very small, then after a relatively short time \( I_{dq} = 0, \hat{r} = \hat{L} = 0 \)
and $\epsilon = 0$; moreover, if $g_T$ is computed in such a way that its time derivative is small, then the effect of the three last terms on the right hand side of (31) can be neglected, yielding

$$g_T V_{\text{Sqd}}^2 \approx \hat{\Theta} z^* + V_{\text{Sqd}}^2 I_{\text{ldq}} - k_p \hat{\eta}.$$  

According to (29) we can approximate $g_T$ as

$$g_T = -k_p \xi - k_p \eta, \quad \hat{\xi} = z, \quad \hat{\eta} = -b \eta + a \hat{z}$$  

(32)

where we have defined $\gamma_2 = \gamma_3 (z^*)^2 / V_{\text{Sqd}}^2$, $k_p = k_p / V_{\text{Sqd}}^2$ and $\xi = \int_0^t \hat{z} \, dt + c_1 - (V_{\text{Sqd}}^2 I_{\text{ldq}}) / (\gamma_3 z^*)^2$ (with $c_1$ an arbitrary constant).

An alternative expressions for this part of the controller is

$$g_T = -\frac{\gamma_3 + k_p \beta}{s^2 + b} z.$$  

(33)

where $p \triangleq d / dt$ is the time derivative operator ($p \rightarrow s$ for the Laplace transform) and $\beta$ represents an approximate of $\hat{z}$ referred in control literature as “dirty derivative.” Finally, expressing the dynamical controller above in the form of a transfer function we get

$$g_T = -\frac{\gamma_3 + k_p \beta}{s^2 + b} z.$$  

(34)

In conclusion, we obtained an adaptive controller composed of (25) with adaptive laws given by (27)–(28) and references $z^* = V_{\text{d}}^2 / 2$ and $I_{\text{dq}}^*$ computed as in (14) with $g_T$ approximated by (33). This controller will be referred in what follows as controller A.

A. An Approximation Useful for Implementation

An alternative expression for the control (25) is

$$E_{\text{dq}} = V_{\text{Sqd}} + K_1 \hat{I}_{\text{dq}} - H \left( I_{\text{dq}}^*, \frac{d}{dt} I_{\text{dq}}^* \right) \hat{\Theta},$$  

where $H(L_{\text{dq}}^*, (dL_{\text{dq}}^*/dt)) \triangleq [L_{\text{dq}}^*, u L_{\text{dq}}^* - (dL_{\text{dq}}^*/dt)]$ and $\hat{\Theta} \triangleq \left[ \hat{F}, \hat{I} \right]^T$.

Let us consider simpler adaptive laws (27)–(28) written in matrix form as

$$\hat{\Theta} = -\Gamma H^T \left( I_{\text{dq}}^*, \frac{d}{dt} I_{\text{dq}}^* \right) \hat{I}_{\text{dq}}$$  

where $\Gamma = \text{diag} \{ \gamma_1, \gamma_2 \}$.

Then, under the assumption that $I_{\text{dq}}^*$ is approximately a constant (let us say $I_{\text{dq}}^* = \bar{T}_{\text{dq}}$), the control can be reduced to

$$E_{\text{dq}} = V_{\text{Sqd}} + K_1 \hat{I}_{\text{dq}} + H \bar{T}_{\text{dq}}^* 0 \right) \Gamma H^T \left( \bar{T}_{\text{dq}}^* 0 \right) \hat{\varphi}$$  

(35)

$$\varphi = \hat{I}_{\text{dq}}$$  

where $H \bar{T}_{\text{dq}}^* 0 \right) \Gamma H^T \left( \bar{T}_{\text{dq}}^* 0 \right)$ is a symmetric positive semidefinite matrix whose computation is very involved. However, for simplicity, and to facilitate the transformation of the controller into stationary frame quantities (as will become clear later), we propose to consider in its place a block diagonal matrix $K_{\text{d}}$ of the form $K_{\text{d}} = \text{diag} \{ k_{\text{d}1}, k_{\text{d}2} \}$, $k_{\text{d}2} = k_{\text{d}2}^T > 0$. We have observed in our simulations that selection of such a $K_{\text{d}}$, disregarding the structure of the matrix $H \bar{T}_{\text{dq}}^* 0 \right) \Gamma H^T \left( \bar{T}_{\text{dq}}^* 0 \right)$ above, does not degrade the performance, but simplifies the implementation considerably. This results in the following slightly more compact expression for the controller:

$$E_{\text{dq}} = V_{\text{Sqd}} + K_1 \hat{I}_{\text{dq}} + K_{\text{d}1} \varphi$$  

(34)

$$\varphi = \hat{I}_{\text{dq}}$$  

(35)

Concerning the control implementation, it is worth noting that the resulting control is expressed in terms of phasor variables, roughly speaking, with the moving average on rotating variables [see (8)]. In order to avoid the possible estimation of such symmetric quantities in the implementation, we express the controllers in terms of stationary frame $\alpha/\beta$ coordinates, this yields

$$e_{\alpha/\beta} = \frac{2}{3} \left( e^{j \omega t} F_{\alpha/\beta} + e^{-j \omega t} F_{\alpha/\beta} \right)$$  

$$= k_2 \dot{\alpha}_\beta + u \dot{\alpha}_\beta + k_3 \left( e^{j \omega t} \varphi^* + e^{-j \omega t} \varphi^* \right)$$  

where $k_2 = (2 / 3) k_{\text{d}1}$, and we have fixed the structure of matrix $K_2$ to be block diagonal of the form $K_2 = \text{diag} \{ k_1, k_2 \}$, $k_2 = k_{\text{d}2}^T > 0$.

Notice that now

$$\ddot{\varphi} = g_T u \dot{\alpha}_\beta - i \dot{L}_{\alpha/\beta}.$$  

(36)

The integral terms are approximated using simple rotations on the rotating-frame quantities (without any moving average), as suggested by [13], relying on the low-pass filtering properties of the existing integrators. This simplifies into equivalent stationary terms all integral rotating-frame terms

$$\varphi^* = e^{-j \omega t} \varphi_{\alpha/\beta}, \quad \varphi^* = e^{-j \omega t} \varphi_{\alpha/\beta},$$

with the symmetry

$$e_{\alpha/\beta} = k_2 \dot{\alpha}_\beta + u \dot{\alpha}_\beta + k_3 (\varphi^* + \varphi^*).$$

This yields

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$$e_{\alpha/\beta} = k_2 \dot{\alpha}_\beta + u \dot{\alpha}_\beta + k_3 \left( e^{j \omega t} \varphi^* + e^{-j \omega t} \varphi^* \right).$$

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Finally, by expressing the dynamical part of the controller in the form of a transfer function we get

$$e_{\alpha/\beta} = k_2 \dot{\alpha}_\beta + u \dot{\alpha}_\beta + k_3 \left( e^{j \omega t} \varphi^* + e^{-j \omega t} \varphi^* \right).$$

(37)

$$\Psi = s^2 + u^2 \dot{\alpha}_\beta$$  

(38)

with the need for a resonant filter at the frequency of the fundamental is evident; this is related to the internal model design concept presented in [14].

This simplified controller formed by (33) and (36)–(38) will be referred as controller B; a block diagram of controller B is shown in Fig. 2.

IV. CONTROLLER DESIGN FOLLOWING A DECOUPLING APPROACH

In this section we explore a simpler adaptive controller which is designed following the PBC approach [8] and assumes that
dc-link voltage and ac current regulation tasks are decoupled. We show that, under certain conditions, this approach results in a control structure which is directly related to the one proposed in Fig. 2. Its structure is described by the interconnection of two control loops, which will be referred to as the “inner” and “outer” control loop.

A. Inner/Current Control Loop

First, notice that the subsystem (11) is indeed a linear system with unknown parameters, which is forced to follow an unknown and slowly varying reference $I_{dq}$. We propose a controller which makes a partial inversion of the system leading to a linear error subsystem to which damping has been added. The error subsystem is disturbed by a parametric error function coming from the unavoidable uncertainty of the inductance value and its parasitic resistance. The error caused by these uncertainties is then compensated by adaptation. As a result of the design process, we obtain the following expression for the controller

$$E_{dq} = V_{Sdq} + K_{pl} \dot{I}_{dq} - \dot{r} I_{dq} - \Delta u \dot{I}_{dq}$$

where $I_{dq}$ is defined by (14), with $g_T$ to be computed later. The estimates $\dot{r}$ and $\dot{L}$ can be computed according to the adaptive laws (27) and (28).

The controller above can also be written in the form of feedback and feedforward terms, where the latter depends on nominal values for the parameters, thus relaxing the control effort of the integral term in the feedback part

$$E_{dq} = V_{Sdq} - r_0 I_{dq} - w_0 \Delta L_{dq}$$

feedback

$$+ K_{pl} \dot{I}_{dq} - \Delta r I_{dq} - \Delta L \Delta \dot{I}_{dq}$$

feedforward

where $L_0$, $r_0$ are the nominal parameters and $(d\Delta r/dt) = \dot{r}$, $(d\Delta L/dt) = \dot{L}$, that comes from the fact that $\dot{r} = r_0 + \Delta r$ and $\dot{L} = L_0 + \Delta L$.

B. Outer/Voltage Control Loop

With the assumption of perfect tracking of the currents (i.e., assuming that the dynamics of the “outer loop” is much slower than that of the “inner control loop”), a control is designed to carry out the output voltage regulation. As will become clear later, this is reduced to the computation of the gain $g_T$ used in the definition of $P_{dq}$ in (14). This “outer control loop” adds damping to reinforce the asymptotic stability of the closed-loop system, and incorporates adaptation via a gradient law to compensate for the uncertainties in $R_f$ and $C$.

We will thus assume that after a relatively short time $\dot{I}_{dq} = 0$, $\dot{r} = r$ and $\dot{L} = L$. Direct substitution of the control (39) in (12) yields the following expression for the $z$ dynamics:

$$\frac{C}{2} \dot{z} = g_T |V_{Sdq}|^2 - V_{Sdq}^T I_{dq} - r |I_{dq}|^2 - \theta \dot{z}$$

where $V_{Sdq}^T I_{dq}$ is indeed a constant. In (40) the term $g_T$ plays the role of the actual control signal. The system (40) can be written in terms of the increments $\dot{z} = z - z^*$ as follows:

$$\frac{C}{2} \dot{z} = g_T |V_{Sdq}|^2 - V_{Sdq}^T I_{dq} - r |I_{dq}|^2 - \theta z^* - \theta \dot{z}.$$ 

To reduce the complexity of the controller we consider the signal $\alpha = V_{Sdq}^T I_{dq} + r |I_{dq}|^2 + \theta z^*$ as an unknown constant.

We propose the following adaptive controller that stabilizes the $z$ dynamics on the desired reference $z^* = V_{Sdq}^2 / 2$. This control contains a damping term and provides adaptation for $\alpha$

$$\frac{C}{2} \dot{z} = - k_2 \dot{z} + \frac{\hat{\alpha}}{|V_{Sdq}|^2}$$

$$\dot{\hat{\alpha}} = - \gamma_3 \dot{z}$$

where the scalars $k_2$, $\gamma_3$ are two positive design parameters and $\hat{\alpha}$ represents the estimate of $\alpha$.

To study the stability of the closed-loop system (40)–(42) we consider an energy function of the form $W_3 = (C/4)\dot{z}^2 + (1/2)\dot{\hat{\alpha}}^2$ (with $\dot{\hat{\alpha}} = \hat{\alpha} - \alpha$), whose time derivative turns out to be negative semidefinite. Again from LaSalle’s arguments we can prove asymptotic stability and convergence of $\dot{\hat{\alpha}}$.

This controller above can also be written in a more familiar form as follows:

$$\frac{C}{2} \dot{z} = - k_2 \dot{z} - k_2 \phi$$

$$\dot{\phi} = \dot{z}$$

where we have defined $k_2 = \gamma_3 / |V_{Sdq}|^2$ and $\phi = -\dot{\hat{\alpha}} / k_2 |V_{Sdq}|^2$.

Equations (39), (43), (44) and (14) with adaptive laws (27) and (28) form a second controller which will be referred to as controller C.

C. Downward Compatibility with a Conventional PI Controller

In this section we show that under certain conditions the controller C presented above is downward compatible with the conventional interconnection of PI controllers used in industrial practice.

First, note that an alternative expression for the control (39) is

$$E_{dq} = V_{Sdq} + K_{pl} \dot{I}_{dq} - H \dot{I}_{dq}$$

Fig. 2. Block diagram of the simplified PBC controller.
where as defined before $\dot{\Theta} = [\dot{\theta}, \dot{\hat{L}}]^T$ and $
abla H(I_{dq}) = [P_{dq}, u\nabla L_{dq}]$. Consider also the following matrix representation of the adaptive laws (27) and (28)

$$\dot{\Theta} = -\Gamma H^T(I_{dq})\hat{I}_{dq}$$

where $\Gamma = \text{diag}\{\gamma_1, \gamma_2\}$

As before, under the assumption that $P_{dq}$ is approximately a constant, the control can be reduced to be

$$E_{dq} = V_{s_{dq}} + K_{p1}\hat{I}_{dq} + K_{i1}\psi, \quad \psi = \hat{I}_{dq}$$

where we have replaced the product $H(I_{dq})\Gamma H^T(I_{dq})$ (a symmetric positive semidefinite matrix) by a block diagonal matrix

$K_{d1} = \text{diag}\{k_{d1}, k_{i1}\}, \quad k_{i1} = k_{f1}^T > 0$.

Another expression for the same controller is

$$E_{dq} = V_{s_{dq}} - r_0P_{dq} - uL_0\nabla L_{dq} + K_{p1}\hat{I}_{dq} + K_{i1}\psi$$

feedback

$$\psi = \hat{I}_{dq}$$

where a feedforward term is introduced to reduce the effort of the integral action; the same controller can be rewritten also as

$$E_{dq} = K_{p1}\hat{I}_{dq} + K_{i1}\psi, \quad \psi = \hat{I}_{dq}$$

where the integral term has absorbed all the constants. As stated before, matrix $K_{d1}$ has been introduced to facilitate the transformation of the controller into stationary frame coordinates.

Let us obtain now an approximate version for the outer loop. For this, assume $(\nu_C + V_d)$ changes very slowly (which, roughly speaking, is equivalent to $P_{dq}$ being almost a constant), the control signal $g_T$ can then be computed as

$$g_T = -K_{f2}\psi_C - K_{p2}\psi, \quad \psi = \hat{\nu}_C$$

where the positive constants $K_{f2} = k_{f2}V_d$ and $K_{p2} = k_{p2}V_d$ have been defined to absorb (approximately) the effect of term $(\nu_C + V_d)/2$.

Thus we have recovered the classical two PI interconnected controllers used to regulate the STATCOM system. Nevertheless, it is our belief that the controllers $A$ and $B$ proposed in previous sections will have a better performance, as their design utilizes more structural informations about the system.

Similar to the previous case, after a sequence of transformations and approximating the moving average (appearing in the integrals) as simple rotations, following the ideas of [13], we can express the PI controller above in terms of the stationary frame coordinates as follows:

$$c_{a\beta} = k_{p2}\hat{\nu}_{a\beta} + k_{i2}\Psi$$

$$\Psi = \frac{s}{s^2 + u_0^2}\hat{\nu}_{a\beta}$$

with $\hat{\nu}_{a\beta} = g_T\psi_0\psi_{a\beta} - i_L\phi_{a\beta}$, and assuming the same block diagonal structure for $K_{d1}$ as stated before.

V. SIMULATION RESULTS

In this section we present the simulation results for the simplified PBC controller $B$ shown in Fig. 2. The configuration we selected to verify the proposed control is based on a VSI switching at 1 kHz, and used as a reactive power and unbalance compensator. Our simulation model includes the switching behavior of the VSI, as modeled in (1). Moreover, the actual gating pulses for VSI switched are generated using space vector modulation (SVM) techniques starting from the $a/b$ average voltage generated by our controller $(c_{a\beta})$. The system parameters, given in per unit, are (see Fig. 1): $L = 0.15$ pu, $r = 0.01$ pu, $C = 0.54$ pu, and $R_L = 200$ pu, with the nominal power 1 MVA and $f = 50$ Hz. Value of $R_L$ accounts only for VSI losses assuming that there is no load connected to capacitor $C$. Control parameters have been selected as follows: $k_1 = 0.397$ pu, $k_2 = 14.42$ pu, $\gamma_0 = 4.23 \cdot 10^{-7}$ pu, $b = 0.92$ pu, and $k_p' = 7.64 \cdot 10^{-6}$ pu. Regarding the selection of control parameters, we used conventional design criteria of decoupled PI control in order to give an initial estimate of control parameter values. For example, damping coefficients $k_1$ have been set equal to the proportional gain of a linear current control with a bandwidth equal to 150 Hz; similar procedure was applied for other parameters. A notch filter tuned to 100 Hz (twice the assumed fundamental) has been used to extract in the control signal the dc component of voltage $\nu_C$ (even during unbalanced conditions).

We first tested the system under balanced conditions by imposing a step variation of the reactive power absorbed by the load at 0.15 s. The load is purely resistive before the transient, and absorbs the maximum reactive current after it. Fig. 3 shows the $a$-$b$-$c$- line currents and their references (dashed lines), Fig. 4 the STATCOM currents and the load currents (dashed lines) and Fig. 5 (from top to bottom) the STATCOM positive sequence $a$-axis current (which represents the reactive power absorbed by VSI), the STATCOM positive sequence $d$-axis current (which represents the active power needed by VSI to compensate for losses), and the dc-link capacitor voltage $\nu_C$. Note that the transient behavior is excellent with a time response which is much shorter than a line period, and there are no steady-state errors. The dc-link voltage $\nu_C$ has a well damped dynamic behavior, and both undershoot and overshoot are bounded and relatively small.

To compare the performance of the proposed controller with conventional solutions, we have firstly implemented a conventional PI control both for the current loops (in $a/b$-coordinates)
Fig. 4. Proposed controller B during a step variation of load reactive power: (solid) STATCOM currents $i$ and (dashed) load currents $i_L$.

Fig. 5. Proposed controller B during a step variation of load reactive power: (from top to bottom) positive sequence $q$-current $I_q$, positive sequence $d$-current $I_d$, and dc-link capacitor voltage $v_c$.

Fig. 6. PI conventional control (designed on $\alpha\beta$-coordinates) during a step variation of load reactive power: (solid) Line currents $i_S$ and (dashed) their references $i_{S*}$.

Fig. 7. PI conventional control (designed on $dq$-coordinates) with unbalanced voltages during a step variation of load reactive power: (top) Line voltages $v_S$ and (bottom) line currents $i_S$.

Fig. 8. Proposed controller B with unbalanced voltages during a step variation of load reactive power: (top) Line voltages $v_S$ and (bottom) line currents $i_S$.

and for the voltage loop. A similar tuning criterion as in the proposed controller has been followed for the conventional PI to obtain comparable results, that is, the current loop bandwidth has been set to 150 Hz; moreover, the feedforward of line voltage has also been applied. The results, reported in Fig. 6, show that the tracking is good only with resistive loads because of voltage feedforward. With any other load the error is quite large due to the limited current loop bandwidth. To solve this problem, the PI control is usually applied to $dq$ coordinates [2]. This solution is very effective in balanced conditions, but it shows some drawbacks with unbalanced line voltages or unbalanced loads. This is shown in Fig. 7 where we repeat the same experiment as above and under the same conditions, but with supply voltage unbalance equal to 10%. Note that the PI control on $dq$ coordinates [2] strongly amplifies the voltage unbalance. In contrast, as shown in Fig. 8, our controller is still able to maintain the overall resistive behavior of the compensated load (i.e., line currents proportional to line voltages) even during unbalanced conditions.
VI. CONCLUSION

The paper describes a family of dissipativity-based controllers for voltage sourced inverters used for reactive power compensation and for three-phase high-quality rectification, operating under possibly unbalanced conditions. The frequency domain modeling of positive sequence and negative sequence components allows a precise tracking with unbalanced supply voltages and with unbalanced current compensation. The passivity-based procedure is explored in terms of damping injection using a dynamic extension, adaptation, and decoupling between the dc-link voltage and ac current regulation, together with some implementation issues. Moreover, a sequence of simplifications that reduce the adaptive controllers to PI structures with feedforward is described. Finally, simulation results illustrate the effectiveness of the proposed methodology.

REFERENCES


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